

Non-Vacuum Bianchi Types I and V in $f(R)$ Gravity

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Abstract

In a recent paper [1], we have studied the vacuum solutions of Bianchi types I and V spacetimes in the framework of metric $f(R)$ gravity. Here we extend this work to perfect fluid solutions. For this purpose, we take stiff matter to find energy density and pressure of the universe. In particular, we find two exact solutions in each case which correspond to two models of the universe. The first solution gives a singular model while the second solution provides a non-singular model. The physical behavior of these models has been discussed using some physical quantities. Also, the function of the Ricci scalar is evaluated.

Keywords: $f(R)$ gravity; Bianchi types I and V .

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1 Introduction

The most interesting phenomenon, which the modern day physics deals with today, is the accelerated expansion of the universe. The explanation for this universe expansion has both theoretical as well as experimental background

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[2]. It has been found that most of the universe contains dark energy and dark matter. The cosmological constant is shown [3] to be an alternative for dark energy. $f(R)$ theory of gravity is an attractive candidate as an alternative theory of gravity in which a general function of the Ricci scalar, $f(R)$, replaces R in the standard Einstein-Hilbert Lagrangian. This provides [4] an easy unification of early time inflation and late time acceleration. The theory also gives a natural gravitational alternative to dark energy.

Lobo [5] gave a brief review of some of the modified theories of gravity that address dark energy and the dark matter problems. In another review [6], Faraoni discussed the explanation of the cosmic acceleration alternative to dark energy in the various versions of $f(R)$ theories of gravity. He investigated the successes of $f(R)$ gravity together with the challenges imposed by minimal criteria for their viability. Sotiriou and Faraoni [7] presented some important aspects of $f(R)$ theories of gravity in Metric, Palatini and Metric-Affine formalisms. They discussed the motivation, actions, field equations, equivalence with other theories, cosmological aspects and constraints, viability criteria and astrophysical applications.

The $f(R)$ theory of gravity is one of the modified theories which is considered most suitable due to cosmologically important $f(R)$ models. These models consist of higher order curvature invariants as functions of the Ricci scalar. Viable $f(R)$ gravity models [8] have been proposed which show the unification of early-time inflation and late-time acceleration. It is hoped that the problem of dark matter can be addressed by using viable $f(R)$ gravity models. Multamäki and Vilja [9] investigated spherically symmetric vacuum solutions in $f(R)$ theory. The same authors [10] also studied the perfect fluid solutions and showed that pressure and density did not uniquely determine $f(R)$. Cognola et al. [11] investigated $f(R)$ gravity at one-loop level in de-Sitter universe. Capozziello et al. [12] explored spherically symmetric solutions of $f(R)$ theories of gravity via the Noether symmetry approach. Recently, Sharif and Kausar [13] studied non-vacuum static spherically symmetric solutions in $f(R)$ gravity.

Hollenstein and Lobo [14] analyzed exact solutions of static spherically symmetric spacetimes in $f(R)$ gravity coupled to non-linear electrodynamics. Azadi et al. [15] studied cylindrically symmetric vacuum solutions in this theory. Momeni and Gholizade [16] extended cylindrically symmetric solutions in a more general way. We have explored static plane symmetric vacuum solutions [17] in $f(R)$ gravity. The field equations are solved using the assumption of constant scalar curvature which may be zero or non-zero.

However, very few attempts have been made so far for solutions with non-constant scalar curvature.

Bianchi types I and V spacetimes play an important role in the study of universe. Lorenz-Petzold [18] studied exact Brans-Dicke Bianchi type I solutions with a cosmological constant. The same author [19] also derived solutions for locally rotationally symmetric (LRS) Bianchi type V space-time in the Brans-Dicke theory of gravitation. It was found that solutions represented anisotropic cosmological models filled with stiff matter and an electromagnetic null field. Berman [20] introduced a different method to solve the field equations by using the variation law of Hubble parameter. The main feature of the variation law is that it gives a constant value of the deceleration parameter. Kumar and Singh [21] investigated perfect fluid solutions using Bianchi type I spacetime in scalar-tensor theory. Singh [22] investigated LRS Bianchi type V cosmology with heat flow in Scalar-Tensor theory. Paul et al. [23] investigated FRW cosmologies in $f(R)$ gravity. In a recent paper [1], we have studied the vacuum solutions of Bianchi types I and V spacetimes in the framework of $f(R)$ gravity.

In this paper, we focuss our attention to explore the perfect fluid solutions of Bianchi types I and V spacetimes in metric $f(R)$ gravity. For this purpose, we take the case of stiff matter to find energy density and pressure of the universe. The paper is organized as follows: In section **2**, we briefly give the field equations in metric $f(R)$ gravity. Sections **3** and **4** are used to find some exact solutions of Bianchi types I and V spacetimes respectively. In the last section, we discuss the results.

2 $f(R)$ Gravity Formalism

The action for $f(R)$ gravity is given by

$$S = \int \sqrt{-g} \left(\frac{1}{16\pi G} f(R) + L_m \right) d^4x, \quad (1)$$

where $f(R)$ is a general function of the Ricci scalar and L_m is the matter Lagrangian. The corresponding field equations are the following:

$$F(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - \nabla_\mu \nabla_\nu F(R) + g_{\mu\nu} \square F(R) = \kappa T_{\mu\nu}, \quad (2)$$

where $F(R) \equiv df(R)/dR$, $\square \equiv \nabla^\mu \nabla_\mu$, ∇_μ is the covariant derivative and $T_{\mu\nu}$ is the standard matter energy-momentum tensor derived from the Lagrangian

L_m . When we contract the field equations, it follows that

$$F(R)R - 2f(R) + 3\Box F(R) = \kappa T. \quad (3)$$

Using this equation in Eq.(2), the field equations take the form

$$F(R)R_{\mu\nu} - \nabla_\mu \nabla_\nu F(R) - \kappa T_{\mu\nu} = \left[\frac{F(R)R - \Box F(R) - \kappa T}{4} \right] g_{\mu\nu}. \quad (4)$$

Thus we have eliminated $f(R)$ from the field equations which helps us to solve the field equations.

3 Exact Bianchi Type I Solutions

In this section we find exact solutions of Bianchi I spacetime in $f(R)$ gravity and some physical quantities.

3.1 Field Equations and Some Physical Quantities

The line element of Bianchi type I spacetime is given by

$$ds^2 = dt^2 - A^2(t)dx^2 - B^2(t)dy^2 - C^2(t)dz^2, \quad (5)$$

where A , B and C are cosmic scale factors. The corresponding Ricci scalar is

$$R = -2\left[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA}\right], \quad (6)$$

where dot means derivative with respect to t . The energy-momentum tensor for perfect fluid yields

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu - pg_{\mu\nu}, \quad (7)$$

satisfying the equation of state

$$p = w\rho, \quad 0 \leq w \leq 1, \quad (8)$$

where ρ and p are energy density and pressure of the fluid while $u_\mu = \sqrt{g_{00}}(1, 0, 0, 0)$ is the four-velocity in co-moving coordinates. Since the metric (5) depends only on t , Eq.(4) yields a set of differential equations for

$F(t)$, A , B , C , ρ and p . Thus the subtraction of the 00-component and 11-component gives

$$-\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{C}\dot{A}}{CA} + \frac{\dot{A}\dot{F}}{AF} - \frac{\ddot{F}}{F} - \frac{\kappa}{F}(\rho + p) = 0. \quad (9)$$

Similarly, we get two more independent equations

$$-\frac{\ddot{A}}{A} - \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{B}\dot{F}}{BF} - \frac{\ddot{F}}{F} - \frac{\kappa}{F}(\rho + p) = 0, \quad (10)$$

$$-\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} + \frac{\dot{C}\dot{F}}{CF} - \frac{\ddot{F}}{F} - \frac{\kappa}{F}(\rho + p) = 0. \quad (11)$$

The conservation equation, $T^{\mu\nu}_{;\nu} = 0$, leads to

$$\dot{\rho} + (p + \rho)\left[\frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right] = 0. \quad (12)$$

Finally, we have four differential equations with six unknowns namely A , B , C , F , ρ and p . The solution of these equations is discussed in the next subsection. In the following we give definition of some physical quantities.

We define the average scale factor and the volume scale factor respectively

$$a = \sqrt[3]{ABC}, \quad V = a^3 = ABC. \quad (13)$$

The generalized mean Hubble parameter H is given in the form

$$H = \frac{1}{3}(H_1 + H_2 + H_3), \quad (14)$$

where $H_1 = \frac{\dot{A}}{A}$, $H_2 = \frac{\dot{B}}{B}$, $H_3 = \frac{\dot{C}}{C}$ are the directional Hubble parameters in the directions of x , y and z axis respectively. Using Eqs.(13) and (14), we obtain

$$H = \frac{1}{3} \frac{\dot{V}}{V} = \frac{1}{3}(H_1 + H_2 + H_3) = \frac{\dot{a}}{a}. \quad (15)$$

The mean anisotropy parameter A is given by

$$A = \frac{1}{3} \sum_{i=1}^3 \left(\frac{H_i - H}{H} \right)^2. \quad (16)$$

The expansion scalar θ and shear scalar σ^2 are defined as follows

$$\theta = u_{;\mu}^{\mu} = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}, \quad (17)$$

$$\sigma^2 = \frac{1}{2}\sigma_{\mu\nu}\sigma^{\mu\nu} = \frac{1}{3}\left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2 - \frac{\dot{A}\dot{B}}{AB} - \frac{\dot{B}\dot{C}}{BC} - \frac{\dot{C}\dot{A}}{CA}\right], \quad (18)$$

where

$$\sigma_{\mu\nu} = \frac{1}{2}(u_{\mu;\alpha}h_{\nu}^{\alpha} + u_{\nu;\alpha}h_{\mu}^{\alpha}) - \frac{1}{3}\theta h_{\mu\nu}, \quad (19)$$

$h_{\mu\nu} = g_{\mu\nu} - u_{\mu}u_{\nu}$ is the projection tensor. In thermodynamics, the entropy of the universe is given by

$$TdS = d(\rho V) + pdV. \quad (20)$$

3.2 Solution of the Field Equations

Subtracting Eqs.(10), (11) and (11) from Eqs.(9), (10) and (9), we get respectively

$$\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} + \frac{\dot{C}}{C}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) + \frac{\dot{F}}{F}\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B}\right) = 0, \quad (21)$$

$$\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} + \frac{\dot{A}}{A}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) + \frac{\dot{F}}{F}\left(\frac{\dot{B}}{B} - \frac{\dot{C}}{C}\right) = 0, \quad (22)$$

$$\frac{\ddot{A}}{A} - \frac{\ddot{C}}{C} + \frac{\dot{B}}{B}\left(\frac{\dot{A}}{A} - \frac{\dot{C}}{C}\right) + \frac{\dot{F}}{F}\left(\frac{\dot{A}}{A} - \frac{\dot{C}}{C}\right) = 0. \quad (23)$$

These equations are exactly the same as given by Eqs.(18)-(20) in [1]. Thus we can write the metric functions explicitly as

$$A = ap_1 \exp[q_1 \int \frac{dt}{a^3 F}], \quad (24)$$

$$B = ap_2 \exp[q_2 \int \frac{dt}{a^3 F}], \quad (25)$$

$$C = ap_3 \exp[q_3 \int \frac{dt}{a^3 F}], \quad (26)$$

where

$$p_1 = (d_1^{-2}d_2^{-1})^{\frac{1}{3}}, \quad p_2 = (d_1d_2^{-1})^{\frac{1}{3}}, \quad p_3 = (d_1d_2^2)^{\frac{1}{3}}, \quad p_1p_2p_3 = 1 \quad (27)$$

and

$$q_1 = -\frac{2c_1 + c_2}{3}, \quad q_2 = \frac{c_1 - c_2}{3}, \quad q_3 = \frac{c_1 + 2c_2}{3}, \quad q_1 + q_2 + q_3 = 0, \quad (28)$$

c_i and d_i are constants of integration. Using power law relation between F and a [1], we have

$$F = ka^m, \quad (29)$$

where k is the constant of proportionality, m is any integer (here taken as 2) and a is given by

$$\begin{aligned} a &= (nlt + k_1)^{\frac{1}{n}}, \quad n \neq 0 \\ a &= k_2 \exp(lt), \quad n = 0, \end{aligned} \quad (30)$$

k_1 and k_2 are constants of integration. It is mentioned here that we have used $H = la^{-n}$, $l > 0$, $n \geq 0$ to get the above equation. Thus we obtain two values of the average scale factor corresponding to two different models of the universe.

3.3 Model of the Universe when $n \neq 0$

For this model, F becomes $F = k(nlt + k_1)^{-\frac{2}{n}}$ and the corresponding metric coefficients A , B and C turn out to be

$$A = p_1(nlt + k_1)^{\frac{1}{n}} \exp\left[\frac{q_1(nlt + k_1)^{\frac{n-1}{n}}}{kl(n-1)}\right], \quad n \neq 1 \quad (31)$$

$$B = p_2(nlt + k_1)^{\frac{1}{n}} \exp\left[\frac{q_2(nlt + k_1)^{\frac{n-1}{n}}}{kl(n-1)}\right], \quad n \neq 1 \quad (32)$$

$$C = p_3(nlt + k_1)^{\frac{1}{n}} \exp\left[\frac{q_3(nlt + k_1)^{\frac{n-1}{n}}}{kl(n-1)}\right], \quad n \neq 1. \quad (33)$$

The mean generalized Hubble parameter and the volume scale factor become

$$H = \frac{l}{nlt + k_1}, \quad V = (nlt + k_1)^{\frac{3}{n}}. \quad (34)$$

The mean anisotropy parameter A turns out to be

$$A = \frac{q_1^2 + q_2^2 + q_3^2}{3l^2 k^2 (nlt + k_1)^{\frac{2}{n}-2}}. \quad (35)$$

The expansion θ and shear scalar σ^2 are given by

$$\theta = \frac{3l}{nlt + k_1}, \quad \sigma^2 = \frac{q_1^2 + q_2^2 + q_3^2}{2k^2(nlt + k_1)^{\frac{2}{n}}}. \quad (36)$$

For stiff matter ($w = 1$), we have $p = \rho$. Thus the energy density and pressure of the universe become

$$2\kappa p = 2\kappa\rho = -\frac{6kl^2}{(nlt + k_1)^{\frac{2}{n}+2}} - \frac{q_1^2 + q_2^2 + q_3^2}{k(nlt + k_1)^{\frac{4}{n}}}. \quad (37)$$

The entropy of universe is given by

$$TdS = \frac{1}{\kappa}[6kl^3(n-2)(nlt + k_1)^{\frac{1}{n}-3} - \frac{l}{k}(q_1^2 + q_2^2 + q_3^2)(nlt + k_1)^{-\frac{1}{n}-1}]. \quad (38)$$

Also, Eq.(12) leads to

$$\rho = \frac{c}{V^2}, \quad (39)$$

where c is an integration constant. It is mentioned here that this value of ρ , when compared with the value obtained in Eq.(37), gives a constraint

$$\kappa c + 3kl^2 = 0 \quad (40)$$

which holds only when $n = 2$ and $q_1^2 + q_2^2 + q_3^2 = 0$. The function of Ricci scalar, $f(R)$ is

$$f(R) = \frac{k}{2}(nlt + k_1)^{\frac{-2}{n}}R + 3kl^2(n-2)(nlt + k_1)^{\frac{-2n-2}{n}}, \quad (41)$$

where $R \equiv R_1 = 6l^2(n-2)(nlt + k_1)^{-2}$. For a special case $n = \frac{1}{2}$, $f(R)$ turns out to be

$$f(R) = \frac{k}{2}\left(-\frac{R}{9l^2}\right)^{\frac{1}{n}} - \frac{9kl^2}{2}\left(-\frac{R}{9l^2}\right)^{\frac{1}{n}+1} \quad (42)$$

which gives $f(R)$ in terms of R .

3.4 Model of the Universe when $n = 0$

Here the metric coefficients take the form

$$A = p_1 k_2 \exp(lt) \exp\left[-\frac{q_1 \exp(-lt)}{klk_2}\right], \quad (43)$$

$$B = p_2 k_2 \exp(lt) \exp\left[-\frac{q_2 \exp(-lt)}{klk_2}\right], \quad (44)$$

$$C = p_3 k_2 \exp(lt) \exp\left[-\frac{q_3 \exp(-lt)}{klk_2}\right]. \quad (45)$$

The directional Hubble parameters H_i and the mean generalized Hubble parameter will become

$$H_i = l + \frac{q_i}{kk_2} \exp(-lt), \quad H = l \quad (46)$$

while the volume scale factor turns out to be

$$V = k_2^3 \exp(3lt). \quad (47)$$

The mean anisotropy parameter A becomes

$$A = \left[\frac{q_1^2 + q_2^2 + q_3^2}{3l^2 k^2 k_2^2} \right] \exp(-2lt) \quad (48)$$

while the quantities θ and σ^2 are given by

$$\theta = 3l, \quad \sigma^2 = \left[\frac{q_1^2 + q_2^2 + q_3^2}{2k^2 k_2^2} \right] \exp(-2lt). \quad (49)$$

For stiff matter, the energy density and pressure turn out to be

$$2\kappa p = 2\kappa \rho = -\frac{6kl^2 \exp(-2lt)}{k_2^2} - \frac{(q_1^2 + q_2^2 + q_3^2) \exp(-4lt)}{kk_2^4}. \quad (50)$$

The corresponding entropy is

$$TdS = \frac{1}{\kappa} [-12kk_2 l^3 \exp(lt) - \frac{l}{kk_2} (q_1^2 + q_2^2 + q_3^2) \exp(-lt)]. \quad (51)$$

The constraint equation with the condition, $q_1^2 + q_2^2 + q_3^2 = 0$, is given by

$$\kappa c - 6kk_2^4 l^2 \exp(4lt) = 0. \quad (52)$$

The function of Ricci scalar, $f(R)$, takes the form

$$f(R) = \frac{k}{2k_2^2} \exp(-2lt) (R - 12l^2) \quad (53)$$

which reduces to

$$f(R) = \sqrt{\frac{3k^3 l^2}{2\kappa c}} [R - 12l^2] \quad (54)$$

using the constraint Eq.(52). This corresponds to the general function $f(R)$ [24],

$$f(R) = \sum a_n R^n, \quad (55)$$

where n may take the values from negative or positive.

4 Exact Bianchi Type V Solutions

Here we shall find exact solutions of the Bianchi type V spacetime.

4.1 Field Equations

The metric for the Bianchi type V spacetime is

$$ds^2 = dt^2 - A^2(t)dx^2 - e^{2mx}[B^2(t)dy^2 + C^2(t)dz^2], \quad (56)$$

where A , B and C are cosmic scale factors and m is an arbitrary constant. The corresponding Ricci scalar is

$$R = -2\left[\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} - \frac{3m^2}{A^2} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA}\right]. \quad (57)$$

With the help of Eq.(4), we can write

$$-\frac{\ddot{B}}{B} - \frac{\ddot{C}}{C} - \frac{2m^2}{A^2} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{C}\dot{A}}{CA} + \frac{\dot{A}\dot{F}}{AF} - \frac{\ddot{F}}{F} - \frac{\kappa}{F}(\rho + p) = 0, \quad (58)$$

$$-\frac{\ddot{A}}{A} - \frac{\ddot{C}}{C} - \frac{2m^2}{A^2} + \frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{B}\dot{F}}{BF} - \frac{\ddot{F}}{F} - \frac{\kappa}{F}(\rho + p) = 0, \quad (59)$$

$$-\frac{\ddot{A}}{A} - \frac{\ddot{B}}{B} - \frac{2m^2}{A^2} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} + \frac{\dot{C}\dot{F}}{CF} - \frac{\ddot{F}}{F} - \frac{\kappa}{F}(\rho + p) = 0. \quad (60)$$

The 01-component can be written by using Eq.(2) in the following form

$$2\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0. \quad (61)$$

We discuss solution of these equations using the same procedure as for the Bianchi type I solutions.

4.2 Solution of the Field Equations

Here we get the same equations Eqs.(21)-(23) as obtained previously with the difference of the constraint equations (using Eq.(61))

$$p_1 = 1, \quad p_2 = p_3^{-1} = P, \quad q_1 = 0, \quad q_2 = -q_3 = Q. \quad (62)$$

Consequently, the metric functions become

$$A = a, \quad B = aP \exp\left[Q \int \frac{dt}{a^3 F}\right], \quad C = aP^{-1} \exp\left[-Q \int \frac{dt}{a^3 F}\right]. \quad (63)$$

4.3 Model of the Universe when $n \neq 0$

The metric coefficients and the directional Hubble parameters are the same as given in [1]. Further, we note that the mean generalized Hubble parameter H , the volume scale factor V and expansion scalar θ turn out to be the same as for the Bianchi type I spacetime while the value of shear scalar σ^2 is

$$\sigma^2 = \frac{Q^2}{k^2(nlt + k_1)^{\frac{2}{n}}}. \quad (64)$$

The mean anisotropy parameter for this model becomes

$$A = \frac{2Q^2}{3l^2k^2(nlt + k_1)^{\frac{2}{n}-2}}. \quad (65)$$

The energy density and pressure of the universe turn out to be

$$\kappa p = \kappa \rho = -\frac{3kl^2}{(nlt + k_1)^{\frac{2}{n}+2}} - \frac{Q^2 + k^2m^2}{k(nlt + k_1)^{\frac{4}{n}}}. \quad (66)$$

The entropy of the universe is given by

$$TdS = \frac{1}{\kappa} [6kl^3(n-2)(nlt + k_1)^{\frac{1}{n}-3} - \frac{2l}{k}(Q^2 + k^2m^2)(nlt + k_1)^{-\frac{1}{n}-1}]. \quad (67)$$

The constraint equation turns out to be same as for the Bianchi type I spacetime with the condition $Q^2 + k^2m^2 = 0$. $f(R)$ is given by

$$f(R) = \frac{k}{2}(nlt + k_1)^{\frac{-2}{n}}R + 3kl^2(n-2)(nlt + k_1)^{\frac{-2n-2}{n}}, \quad (68)$$

where $R = 6l^2(n-2)(nlt + k_1)^{-2} + 8m^2(nlt + k_1)^{\frac{-2}{n}}$. For a special case, $n = \frac{1}{2}$, it follows that

$$\begin{aligned} f(R) &= \frac{k}{2} \left[\frac{-9l^2 \pm \sqrt{81l^4 + 32m^2R}}{2R} \right]^{-2} R \\ &\quad - \frac{9kl^2}{2} \left[\frac{-9l^2 \pm \sqrt{81l^4 + 32m^2R}}{2R} \right]^{-3} \end{aligned}$$

which gives $f(R)$ in terms of R only.

4.4 Model of the Universe when $n = 0$.

Here the metric coefficients, the mean generalized Hubble parameter, volume scale factor and expansion scalar are the same as given for Bianchi type I spacetime. However, the value of shear scalar σ^2 is

$$\sigma^2 = \frac{Q^2}{k^2 k_2^2} \exp(-2lt), \quad (69)$$

and the mean anisotropy parameter A here becomes

$$A = \frac{2Q^2 \exp(-2lt)}{3l^2 k^2 k_2^2}. \quad (70)$$

The energy density and pressure of the universe are

$$\kappa p = \kappa \rho = -\frac{3kl^2 \exp(-2lt)}{k_2^2} - \left(\frac{(Q^2 + m^2 k^2)}{k k_2^4} \right) \exp(-4lt). \quad (71)$$

The entropy is

$$TdS = \frac{1}{\kappa} \left[-12k k_2 l^3 \exp(lt) - \frac{2l}{k k_2} (Q^2 + m^2 k^2) \exp(-lt) \right]. \quad (72)$$

The constraint equation with the condition, $Q^2 + m^2 k^2 = 0$, turns out to be same as given by Eq.(52). The function of Ricci scalar, $f(R)$, takes the form

$$f(R) = \frac{k}{2k_2^2} \exp(-2lt) (R - 12l^2) \quad (73)$$

with $R = -12l^2 + \left(\frac{8m^2}{k_2^2} \right) \exp(-2lt)$. Here we can get $f(R)$ in terms of R

$$f(R) = \frac{k}{16m^2} [R^2 - 144l^4] \quad (74)$$

which also corresponds to the general function $f(R)$ given by Eq.(55).

5 Summary and Conclusion

This paper is devoted to study the universe expansion in metric $f(R)$ gravity. We have found exact solutions of the Bianchi types I and V spacetimes

using the non-vacuum field equations. These solutions correspond to two models of the universe, i.e., a singular model and a non-singular model. We have evaluated some important cosmological physical quantities for these solutions such as expansion scalar θ , shear scalar σ^2 and mean anisotropy parameter A . The entropy of the universe is also found. We would like to mention here that solutions for both the spacetimes correspond to perfect fluid [25, 26] in GR. It is found that the general function $f(R)$ includes squared power of the Ricci scalar for the non-singular model.

Firstly we investigate singular model ($n \neq 0$) of the universe which has singularity at $t \equiv t_s = -\frac{k_1}{m}$. The physical parameters H_1 , H_2 , H_3 , H , θ , and σ^2 are all infinite at this point for $n > 0$ but the volume scale factor vanishes. The mean anisotropy parameter A is also infinite at this point for $0 < n < 1$ and it will vanish for $n > 1$. The function of the Ricci scalar, energy density, pressure and T are also infinite while the metric functions vanish at this point of singularity. The model also suggests that the expansion and shear scalar decrease for $n > 0$ with the passage of time. The mean anisotropy parameter also decreases for $n > 1$ with the increase in time. This indicates that after a large time the expansion will stop completely and the universe will achieve isotropy. The isotropy condition, i.e., $\frac{\sigma^2}{\theta} \rightarrow 0$ as $t \rightarrow \infty$, is also satisfied. The entropy of the universe is infinite for $n > \frac{1}{3}$. Moreover, the second law of thermodynamics is valid for $n > 2$ with the conditions $q_1^2 + q_2^2 + q_3^2 = 0$ and $Q^2 + k^2 m^2 = 0$. Thus we can conclude from these observations that the model starts its expansion from zero volume with infinite energy density and pressure at $t = t_s$ and it continues to expand with time.

The second model is non-singular model ($n = 0$) of the universe. The physical parameters H_1 , H_2 , H_3 , σ^2 and A are all finite for all finite values of t . The mean generalized Hubble parameter H and expansion scalar θ is constant while $f(R)$ is also finite here. The metric functions do not vanish for this model. The entropy of the universe is finite while the second law of thermodynamics is valid for $kk_2 < 0$ with the conditions $q_1^2 + q_2^2 + q_3^2 = 0$ and $Q^2 + k^2 m^2 = 0$. The energy density and pressure become infinite as $t \rightarrow -\infty$ which shows that the universe started its evolution in an infinite past with a strong pressure and energy density. The isotropy condition is also verified for this model. The volume scale factor increases exponentially with time which indicates that the universe starts its expansion with zero volume from infinite past.

We would like to mention here that qualitative analysis of the solutions found has not been done. However, it would be interesting to perform simu-

lation procedure for the consistency of the results with Wilkinson Microwave Anisotropy Probe (WMAP) data as given in the literature.

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